RADIATIVE HEAT FLUX IN THE HEATING OF A SOLID BODY BY A SOURCE WITH VARIABLE TEMPERATURE

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The variation of the heat flux at the surface of a solid body heated in a medium with variable temperature is analyzed. A functional representation in terms of dimensionless groups is proposed. The proposed relations are checked experimentally.

In heating practice one often encounters processes involving variable-temperature sources.

The diffusion of heat in an isotropic solid is described by the following system of differential equations

$$\frac{\partial \theta(\zeta, \mathrm{Fo})}{\partial \mathrm{Fo}} = \nabla^2\theta(\zeta, \mathrm{Fo}), \tag{1}$$

$$\frac{\partial \theta(1, \mathrm{Fo})}{\partial \zeta} = \mathrm{Ki}_{\max}[\theta_m^4(\mathrm{Fo}) - \theta_s^4(\mathrm{Fo})], \tag{2}$$

$$\frac{\partial \theta(0, \mathrm{Fo})}{\partial \zeta} = 0, \tag{3}$$

$$\theta(\zeta, 0) = 0. \tag{4}$$

Here

$$\nabla^2 = \frac{\partial^2}{\partial \zeta^2} + \frac{\xi - 1}{\zeta}\frac{\partial}{\partial \zeta},$$

and the values of ζ and ξ for the three classical body shapes are given in Table 1. The following cases, in which the temperature of the heat source varies with time according to a linear or an exponential law, are of considerable interest:

$$\theta_m(\mathrm{Fo}) = \theta_{mo} + \mathrm{Pd\,Fo}, \tag{a}$$

$$\theta_m(\mathrm{Fo}) = 1 - (1 - \theta_{mo})\exp(-\mathrm{Pd\,Fo}). \tag{b}$$

The rigorous solution of (1)-(4) taking into account (a) or (b) would be quite difficult due to the nonlinearity of boundary condition (2). For this reason the temperature field of this problem is usually determined by approximate methods [2-5].

The problem can be solved by finite differences. The body is divided into several layers $\Delta x = R/N$, and the differential equations (1)-(4) are replaced by finite-difference equations [7-9].

For the plate

$$\theta_{s,\ \mathrm{Fo}+\Delta \mathrm{Fo}} = \theta_{s,\ \mathrm{Fo}} + 2\,\frac{\mathrm{Ki}_{\max}}{N}\,\frac{\Delta \mathrm{Fo}}{\Delta X^2}\,[\theta^4_{m,\ \mathrm{Fo}} - \theta^4_{s,\ \mathrm{Fo}}] - \tag{5}$$

$$-2\,\frac{\Delta \mathrm{Fo}}{\Delta X^2}\,(\theta_{s,\,\mathrm{Fo}}-\theta_{N-1,\,\mathrm{Fo}}),$$

$$\theta_{n,\,\mathrm{Fo}+\Delta\,\mathrm{Fo}}=\theta_{n,\,\mathrm{Fo}}+\frac{\Delta \mathrm{Fo}}{\Delta X^2}\,(\theta_{n+1,\,\mathrm{Fo}}-2\theta_{n,\,\mathrm{Fo}}+\theta_{n-1,\,\mathrm{Fo}}),$$

$$\theta_{0,\,\mathrm{Fo}+\Delta\,\mathrm{Fo}}=\theta_{0,\,\underset{s}{\mathrm{Fo}}}+2\,\frac{\Delta \mathrm{Fo}}{\Delta X^2}\,(\theta_{1,\,\mathrm{Fo}}-\theta_{0,\,\mathrm{Fo}}).$$

For the cylinder

$$\theta_{s,\,\mathrm{Fo}+\Delta\,\mathrm{Fo}}=\theta_{s,\,\mathrm{Fo}}+2\,\frac{\mathrm{Ki}_{\max}}{N-\frac{1}{2}}\,\frac{\Delta \mathrm{Fo}}{\Delta\rho^2}\,[\theta^4_{m\mathrm{Fo}}-\theta^4_{s,\,\mathrm{Fo}}]-$$

$$-2\,\frac{(2-2/N)}{(2-1/N)}\,\frac{\Delta \mathrm{Fo}}{\Delta\rho^2}\,(\theta_{s,\,\mathrm{Fo}}-\theta_{N-1,\,\mathrm{Fo}}),$$

$$\theta_{n,\,\mathrm{Fo}+\Delta\,\mathrm{Fo}}=\theta_{n,\,\mathrm{Fo}}+\frac{\Delta \mathrm{Fo}}{\Delta\rho^2}\,\left[(\theta_{n+1,\,\mathrm{Fo}}-\theta_{n,\,\mathrm{Fo}})\left(1+\frac{\Delta r}{2r_n}\right)-\right. \tag{6}$$

$$\left.-(\theta_{n,\,\mathrm{Fo}}-\theta_{n-1,\,\mathrm{Fo}})\left(1-\frac{\Delta r}{2r_n}\right)\right],$$

$$\theta_{0,\,\mathrm{Fo}+\Delta\,\mathrm{Fo}}=\theta_{0,\,\mathrm{Fo}}+4\,\frac{\Delta \mathrm{Fo}}{\Delta\rho^2}\,(\theta_{1,\,\mathrm{Fo}}-\theta_{0,\,\mathrm{Fo}}).$$

The thermophysical properties are assumed to be constant, and the initial temperature distribution is assumed to be uniform over the cross section of the body.

Table 1
Values of ξ, ζ, and $H(\mu_n)$ for the Three Classical Body Shapes

Body	ξ	ζ	$H(\mu_n)$	Characteristic Equation
Plate	1	$\frac{x}{R}$	$(-1)^n \cos\mu_n \frac{x}{R}$	$\sin\mu_n = 0$
Cylinder	2	$\frac{r}{R}$	$\frac{I_0\left(\mu_n \frac{r}{R}\right)}{I_0(\mu_n)}$	$I_0'(\mu_n) = 0$
Sphere	3	$\frac{r}{R}$	$\frac{\sin\mu_n \frac{r}{R}}{\frac{r}{R}\sin\mu_n}$	$\mathrm{tg}\,\mu_n = \mu_n$

The half-thickness of the plate (in the case of symmetrical heating on both sides) and the radius of the cylinder were divided into 20 layers of equal thickness. The ratio between the time and space steps was

$$\frac{\Delta \mathrm{Fo}}{\Delta X^2}=\frac{\Delta \mathrm{Fo}}{\Delta\rho^2}=\frac{1}{4},$$

i.e., the step in the Fourier number was $\Delta\mathrm{Fo} = 1/1600$.

Calculations of radiative heat transfer were performed on electronic digital computers over wide ranges of dimensionless initial temperature of the solid body $\theta_0 = 0.15$-0.5, dimensionless initial temperature of the heating medium $\theta_{mo} = 0.4$-0.8, and radiative heat-transfer parameter $\mathrm{Ki}_{max} = 0.5$-10 for the linear (Pd = 0.05-0.150) and exponential (Pd = 0.5-1.0) laws.

In the majority of cases the calculation was carried out up to the value Fo = 10.

The large amount of numerical material obtained on digital computers makes it possible to follow the variation of the radiative heat flux during the heating of solid bodies by variable-temperature sources.

An analysis of the heat-flux curves shows that these can be satisfactorily represented by the relatively simple relations

$$Q_a = \frac{q_a(\mathrm{Fo})}{q_m(\mathrm{Fo})} = \exp(-d^2), \tag{7}$$

$$Q_s = \frac{q_s(\mathrm{Fo})}{q_m(\mathrm{Fo})} = 1 - \exp(-d^2), \tag{8}$$

where

$$d = d_0 + \xi \mathrm{Ki}^m \mathrm{Fo}.$$

From (8) we obtain

$$T_s(\mathrm{Fo}) = T_m(\mathrm{Fo}) \sqrt[4]{1 - \exp(-d^2)}. \tag{9}$$

Figure 1 represents the results of (7) for the case of a flat plate.

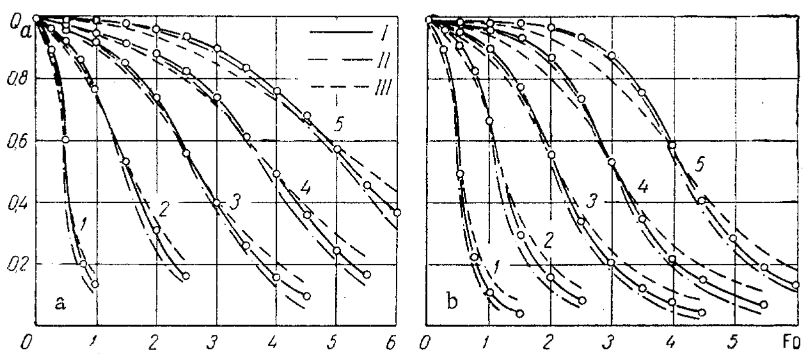

Fig. 1. Variation of radiative heat flux at the surface of an infinite plate: a) linear variation of heating-medium temperature with time [θ_0 = 0.15, θ_{mo} = 0.4, Pd = 0.15, m = 1.2, (1) Ki_{mo} = 0.64, (2) 0.32, (3) 0.128, (4) 0.064, (5) 0.032,] b) exponential variation of heating-medium temperature [θ_0 = 0.15, θ_{mo} = 0.6, Pd = 1.0, m = 1.2, (1) Ki_{mo} = 2.16, (2) 1.08, (3) 0.432, (4) 0.216, (5) 0.108]: I) computer data, II) zonal method [5], III) according to (7).

Thus, in quite a number of cases involving similar heating conditions it is advisable to use the approximate, but simpler, formula for the heat flux.

Using the approximate formula for the heat flux at the surface (7), one can calculate the temperature field throughout the body, which can be represented in integral form [6]

$$T(\zeta, \mathrm{Fo}) = T_0 + \xi \frac{R}{\lambda} \int_0^{\mathrm{Fo}} q_a(\mathrm{Fo})\, d\,\mathrm{Fo} + \frac{2R}{\lambda} \sum_{n=1}^{\infty} H(\mu_n) \exp(-\mu_n^2 \mathrm{Fo}) \int_0^{\mathrm{Fo}} q_a(\mathrm{Fo}) \exp(\mu_n^2 \mathrm{Fo})\, d\,\mathrm{Fo}. \tag{10}$$

The expressions for $H(\mu_n)$ and the characteristic equations are given in Table 1. Equation (8), which approximates the radiative heat flux emitted by the surface of the body, agrees with numerical results to within ~ 14%.

Table 2 represents the errors associated with the temperature-field approximations (9), (10) for various heating-medium parameters. Results are given for the surface and center-plane of an infinite plate of thickness 2R = 200 mm and

$$\lambda = 45.36\ \text{W/m} \cdot \text{deg},\ a = 0.045\ \text{m}^2/\text{hr},\ \varepsilon_r C_0 = 4.65\ \text{W/m}^2 \cdot \text{deg}^4$$

Table 2

Values of Temperature at the Surface and Center-Plane of an Infinite Plate During Heating

t, hr	T_s, °K		δ, %	T_s, °K		δ, %
	Computer Results	According to (9)		Computer Results	According to (10)	
			$T_m(\tau) = 1073 + 130\tau$			
0.5	686.3	678.7	+1.11	517.5	512.3	+0.89
1.0	1068.4	1021.3	+4.40	813.4	778.8	+4.12
1.5	1174.7	1223.6	+4.16	1052.0	1012.3	+3.74
2.0	1338.1	1373.5	—2.62	1156.9	1184.1	—2.47
2.5	1443.2	1460.9	—1.78	1284.1	1358.2	—1.58
3.0	1462.2	1471.4	—0.55	1396.3	1407.5	—0.54
			$T_m(\tau) = 1073 + 320\tau$			
0.5	713.8	734.2	—2.90	607.5	623.7	—2.77
1.0	1199.3	1260.7	—4.98	932.2	974.2	—4.59
1.5	1483.6	1513.5	—2.02	1231.7	1254.3	—1.96
2.0	1680.2	1698.9	—1.07	1564.3	1580.1	—0.93

One can see that the error in the temperature field does not exceed 5%. Expressions (7)-(9) are recommended for those values of the dimensionless groups, which cover the region of initial data for computer calculations.

These limits include a wide range of cases of industrial interest.

Experimental check. A spherical body was heated in a spherical furnace with a 1.2-mm nichrome heater. Furnace power was ~ 1.5 kW. Two hemispherical copper screens were used to equalize temperatures. Platinum/platinum-rhodium thermocouples of 0.5 mm diam. were used to measure temperatures inside the body. Temperatures were recorded by means of a 20-point electronic potentiometer. The variation of the heater temperature was controlled by an automatic control system. The signal from a 0.5-mm chromel-alumel thermocouple was fed to a programmed electronic controller. The heating device was installed in a vacuum chamber. A moderate-vacuum diffusion pump was used.

Figure 2 shows the variation of the surface temperature and of the maximum temperature difference in the body for heating of a 100-mm sphere of St3 steel by a heater with exponentially varying temperature. The body was heated in a vacuum of ~ 1.0667×10^{-3}-6.667×10^{-3} newton/m^2.

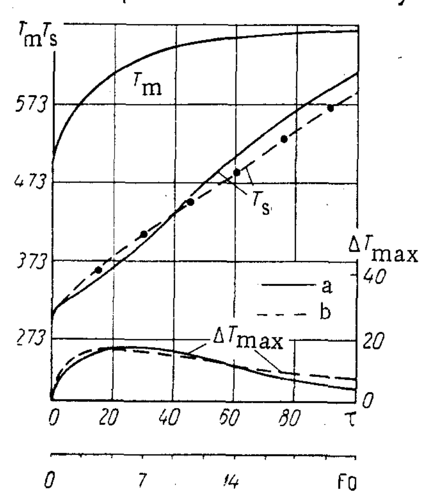

Fig. 2. Heating of a 100-mm steel sphere: a) experimental, b) calculated from (9), (10).

The experimental data were compared with the results of calculations according to (9), (10) with $\varepsilon_r = 0.75$, $\lambda = 45.4$ W/m · deg, $a = 0.035$ m^2/hr/11.

The reduced absorptivity was calculated by Christiansen's formula [10]

$$\frac{1}{\varepsilon_r} = \frac{1}{\varepsilon_1} + \left(\frac{R_1}{R_2}\right)^2\left(\frac{1}{\varepsilon_2} - 1\right),$$

where R_1 is the radius of the heated body, and R_2 is the inside radius of the spherical furnace. In our case $R_1 = 0.05$ m, $R_2 = 0.0625$ m, $\varepsilon_1 = 0.88$ (black oxidized copper), $\varepsilon_2 = 0.8$ (oxidized steel) [11].

The thermophysical properties used were the integral means

$$\lambda = \frac{\int_{T_0}^{T_f} \lambda(T)\,dT}{T_0 - T_f}, \quad C = \frac{\int_{T_0}^{T_f} C(T)\,dT}{T_0 - T_f},$$

where T_f is the temperature at the end of heating averaged over the body cross section. We used the surface-mean temperature, since at the end of the heating process it differed very little from the cross section mean (the temperature used was the surface temperature at the end of the experiment).

In most technological applications, the final value of surface temperature is specified. If it is not specified, then the functional dependence of the thermal coefficients may be represented by an equivalent stepwise-varying function.

It should be noted that for almost linear variation of λ and C the arithmetical mean can be substituted for the integral mean.

The proposed relations (7)-(9) are in satisfactory agreement with experiment and may be used for quick calculation of the heating of solid bodies by radiation from a variable-temperature heater.

NOTATION

ζ – generalized dimensionless coordinate; ξ – body shape factor; $\theta - T(x, \tau)/T_{max}$ – dimensionless absolute temperature, equal to ratio of the temperature at a given point and time to the final temperature of the medium; $Pd = bR^2/aT_{max}$, $Pd = kR^2/a$ – Predvoditelev numbers for the linear and exponential laws, respectively; $Ki_{max} = q_{max} R/\lambda T_{max}$, $Ki_{mo} = q_{mo}R = \lambda T_{mo}$, $Ki = q_m(\tau)R/\lambda T_m(\tau)$ – Kirpichev numbers for the maximum, initial, and time-dependent temperature of the heater, respectively; $q_m(\tau) = \varepsilon_r C_0 (T_m(\tau)/100)^4$ – radiative heat flux from heater to body and from body surface to heater, respectively; $q_a(\tau)$ – heat absorbed by body during heating; $Q_a = q_a(\tau)/q_m(\tau)$ – dimensionless net absorbed heat flux; $Q_s = q_s(\tau)/q_m(\tau)$ – dimensionless heat flux from body surface to heater; m – exponent; d_0 – constant defined by (8) at Fo = 0; ε_r – reduced absorptivity of system.

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